

# Inversion Theorem of Fractional Sumudu–Fourier Transform

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## ABSTRACT

The Fractional Sumudu–Fourier Transform (FrSFT) is a useful integral transform that combines the important features of the Sumudu Transform and the Fractional Fourier Transform. It provides an effective approach for studying functions with fractional and oscillatory behaviour and has potential applications in mathematics, signal processing, and engineering. An inversion theorem is an essential property of any integral transform because it guarantees that the original function can be recovered from its transformed form. In this work, the inversion theorem for the Fractional Sumudu–Fourier Transform is established using testing function spaces and generalized functions. The obtained result confirms the mathematical validity of the transform and provides a foundation for its future applications in both theoretical and applied mathematics.

**Keywords:** Fractional Sumudu–Fourier Transform (FrSFT), Sumudu Transform, Fourier Transform, Inversion Theorem, Testing Function Space, Generalized Functions.

## 1 Introduction

Integral transforms are widely used to simplify complex equations in mathematics, physics, and engineering. Among them, the Fourier transform plays a key role by converting functions into the frequency domain, while its inversion theorem helps recover the original form. These ideas form the foundation for studying the Fractional Sumudu–Fourier Transform [1], [2].

The Fractional Sumudu–Fourier Transform combines the strengths of the Sumudu and Fractional Fourier transforms. It enhances flexibility in signal representation and provides effective tools for solving fractional differential equations, with applications in signal processing and applied mathematics [3]. By introducing fractional order, the Fractional Fourier Transform allows improved time–frequency analysis. When combined with the Sumudu transform, it creates a powerful framework for studying generalized functions and solving complex fractional systems efficiently [4].

The Sumudu transform preserves physical units, while the Fractional Fourier Transform offers flexible domain representation. Their combination results in a practical and efficient method for solving fractional equations and analysing signals [5]. This hybrid transform simplifies mathematical modelling and provides a reliable approach for handling fractional systems. It is particularly useful in applied mathematics and engineering problems involving complex behaviours [6]. The Fractional Sumudu–Fourier Transform offers a balanced approach between analytical simplicity and computational efficiency, making it suitable for studying fractional differential equations and real-world applications [7].

It provides a structured framework for time–frequency analysis and helps in understanding generalized mathematical functions with applications in physics and signal processing [8]. Due to its flexibility and efficiency, this transform has become a valuable tool for solving advanced mathematical problems and analysing complex systems [9], [10]. The Fractional Sumudu Transform extends the classical version using fractional calculus, offering improved flexibility for modelling and solving differential equations in science and engineering [11], [12].

The Sumudu transform itself is known for preserving units and simplifying calculations. Its fractional form enhances its capability to model systems with memory and hereditary properties [13], [14]. Because of its simple

structure and effectiveness, the fractional version is widely applied in engineering, physics, and applied mathematics [15], [16]. It is based on fractional calculus concepts and shows a strong connection with the Laplace transform, making it useful for solving fractional differential equations [17].

The transform is also effective in handling fractional integrals and derivatives, especially in solving diffusion-type problems [18]. With its unit-preserving and scale-consistent properties, the Sumudu transform is closely related to the Laplace transform and is widely used in various scientific fields [19]. Overall, the fractional Sumudu transform provides a flexible and efficient framework for analysing systems with fractional behaviour and solving complex mathematical problems [20].

This paper is organized as follows: Section 1, presents an introduction to integral transforms, including the Sumudu transform, Fourier transform, and their fractional extensions. Section 2, discusses the definitions of Fractional Sumudu – Fourier transform. Section 3, describe the suitable testing function space. Section 4, present the Distributional Fractional Sumudu – Fourier Transform and discusses its formulation. Section 5, focuses on the inversion theorem and related theoretical results. Sections 6 explain the uniqueness theorem, while Section 7 concludes the paper with applications, observations, and potential for future research directions [1], [21].

## 2 Definition of Fractional Sumudu-Fourier Transform

The Fractional Sumudu–Fourier transform of  $f(t, x)$  is given by,

$$FrSFT \{f(t, x)\} = F_\alpha(s, p) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t, x) K_\alpha(t, x, s, p) dt dx$$

$$FrSFT \{f(t, x)\} = \frac{1}{s} \sqrt{\frac{1 - i \cot \alpha}{2\pi i}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t, x) e^{\frac{1}{2}[(t^2 + \frac{1}{s^2}) \cot \alpha - \frac{2t}{s} \operatorname{cosec} \alpha]} e^{-ipx} dt dx$$

where,

$$K_\alpha(t, x, s, p) = \frac{1}{s} \sqrt{\frac{1 - i \cot \alpha}{2\pi i}} e^{\frac{1}{2}[(t^2 + \frac{1}{s^2}) \cot \alpha - \frac{2t}{s} \operatorname{cosec} \alpha]} e^{-ipx}$$

is the kernel of Fractional Sumudu–Fourier transform such that  $0 < \alpha < \frac{\pi}{2}$ .

$K_\alpha(t, x, s, p)$  belongs to the testing function space and  $f(t, x)$  lies in its dual space.

## 3 Testing Function Space $E(R^n)$

An infinitely differentiable complex valued smooth function  $\phi(t, x)$  on  $R^n$  belongs to  $E(R^n)$ , if for each compact set  $I \subset S_{a,b}$  such that,

$$S_{a,b} = \{t, x: t, x \in R^n, |t| \leq a, |x| \leq b, a > 0, b > 0\}$$

and the seminorm,  $\gamma_{E,l,n}(\phi) = \sup_{t, x \in I} |D_t^l D_x^n \phi(t, x)| < \infty$  where  $l, n \geq 0$

Hence,  $E(R^n) = \{\phi \in R^n: \gamma_{E,l,n}(\phi) < \infty, \forall l, n \geq 0\}$

Moreover,  $E(R^n) = \{\phi \in R^n: \operatorname{supp}(\phi) \subseteq S_{a,b}\}$

where  $\operatorname{supp}(\phi)$  denotes the support of function  $\phi$ .

Finally, the dual space of  $E(R^n)$  is defined by,

$$E^*(R^n) = \{T: E(R^n) \rightarrow C; T \text{ is the continuous linear functional}\}.$$

A generalized function  $f$  is said to be Fractional Sumudu–Fourier transformable if  $f \in E^*(R^n)$ .

## 4 Distributional Fractional Sumudu-Fourier transform

The distributional Fractional Sumudu–Fourier transform of  $f(t, x) \in E(R^n)$  defined by,

$$FrSFT \{f(t, x)\} = F_\alpha(s, p) = \langle f(t, x), K_\alpha(t, x, s, p) \rangle$$

where,

$$K_{\alpha}(t, x, s, p) = \frac{1}{s} \sqrt{\frac{1 - i \cot \alpha}{2\pi i}} e^{\frac{1}{2}[(t^2 + \frac{1}{s^2}) \cot \alpha - \frac{2t}{s} \operatorname{cosec} \alpha]} e^{-ipx}$$

for each fixed  $t$  ( $-\infty < t < \infty$ ),  $x$  ( $-\infty < x < \infty$ ),  $s > 0$ ,  $p > 0$  and  $0 < \alpha < \frac{\pi}{2}$ .

Also denotes  $A_{\alpha} = \sqrt{\frac{1 - i \cot \alpha}{2\pi i}}$  and  $B_{\alpha} = \frac{1}{2 \sin \alpha}$

Therefore, the above kernel can be written as,

$$K_{\alpha}(t, x, s, p) = \frac{1}{s} A_{\alpha} e^{B_{\alpha}[(t^2 + \frac{1}{s^2}) \cos \alpha - \frac{2t}{s}]} e^{-ipx}$$

## 5 Inversion Theorem of Fractional Sumudu-Fourier Transform

### 5.1 Lemma 1

Statement: Let  $FrSFT\{f(t, x)\} = F_{\alpha}(s, p)$  for  $0 < \alpha \leq \frac{\pi}{2}$  and  $\operatorname{supp} f \subset S_{a,b}$  such that

$$S_{a,b} = \{t, x: t \in \mathbb{R}^n, |t| \leq a, |x| \leq b, a > 0, b > 0\}$$

Let  $\phi(t, x) \in D(t)$  and define

$$\psi(s, p) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(t, x) \overline{K}(t, x, s, p) dt dx$$

then for any fixed real number  $r, b$  with  $-\infty < r < \infty$  and  $-\infty < b < \infty$ .

$$\int_{-r}^r \int_{-b}^b \langle f(t, x), K_{\alpha}(t, x, s, p) \rangle \psi(s, p) d\tau d\mu = \left\langle f(t, x), \int_{-r}^r \int_{-b}^b K_{\alpha}(t, x, s, p) \psi(s, p) d\tau d\mu \right\rangle \dots\dots\dots (5.1)$$

where  $s = \sigma + i\tau$  and  $p = \lambda + i\mu$

where  $s, p \in \mathbb{C}^n$  and  $t, x$  are restricted a compact subset of  $\mathbb{R}^n$ .

Also  $\sigma$  and  $\lambda$  are fixed such that  $\sigma_1 < \operatorname{Re} t < \sigma_2$  and  $\lambda_1 < \operatorname{Re} x < \lambda_2$

**Proof:** For  $\phi(t, x) \equiv 0$  then the result is trivial. So assume that  $\phi(t, x) \neq 0$ .

If  $FrSFT\{f(t, x)\} = F_{\alpha}(s, p)$  then the  $F_{\alpha}(s, p)$  is analytic for  $s > 0$  and  $\psi(s, p)$  is an entire function. Therefore, it can be easily seen that,

$$\int_{-r}^r \int_{-b}^b K_{\alpha}(u, v, s, p) \psi(s, p) d\tau d\mu$$

Where,  $s = \sigma + i\tau$  and  $p = \lambda + i\mu$  is a  $C^{\infty}$  function of  $u$  and  $v$  which is belongs to  $E$ . Hence the R.H.S. of (5.1) is meaningful.

To prove the equality, we construct the Riemann-sums for this integral and write

$$\int_{-r}^r \int_{-b}^b \langle f(u, v), K_{\alpha}(u, v, s, p) \rangle \psi(t, x) d\tau d\mu$$

since the function  $G(s, p) = \langle f(u, v), K_{\alpha}(u, v, s, p) \rangle \psi(t, x)$  is continuous on the compact rectangle  $[-r, r] \times [-b, b]$ .

It is Riemann integrable. Therefore, by the definition of the double Riemann integral, the integral can be represented as the limit of its corresponding double Riemann sum over partition of the rectangle. Choosing a sample point as  $(\sigma + i\tau_{hg}, \lambda + i\mu_{jk})$  in each rectangle then we obtain,

$$\lim_{g,k \rightarrow \infty} \sum_{h \rightarrow -g}^{g-1} \sum_{j \rightarrow -k}^{k-1} \langle f(u, v), K_{\alpha}(u, v, \sigma + i\tau_{hg}, \lambda + i\mu_{jk}) \rangle \psi(\sigma + i\tau_{hg}, \lambda + i\mu_{jk}) \Delta\tau_{hg} \Delta\mu_{jk}$$

since the inner product is linear in its second argument and the above summation is finite for each partition. The finite sum may be brought inside the inner product. Hence,

$$\lim_{g,k \rightarrow \infty} \left\langle f(u, v), \sum_{h \rightarrow -g}^{g-1} \sum_{j \rightarrow -k}^{k-1} K_{\alpha}(u, v, \sigma + i\tau_{hg}, \lambda + i\mu_{jk}) \psi(\sigma + i\tau_{hg}, \lambda + i\mu_{jk}) \Delta\tau_{hg} \Delta\mu_{jk} \right\rangle$$

we show that the last summation converges in  $E$  to the integral on R.H.S. of equation (5.1).

Therefore, the convergence of sequence in the testing function space  $E$  is determined by its associated family of seminorm  $\gamma_{E,l,n}$ . Therefore, is sufficient to estimate the seminorm of the difference between Riemann-sums and the corresponding integral. Therefore, we write,

$$\gamma_{E,l,n} \left\{ \sum_{h \rightarrow -g}^{g-1} \sum_{j \rightarrow -k}^{k-1} K_{\alpha}(u, v, \sigma + i\tau_{hg}, \lambda + i\mu_{jk}) \psi(\sigma + i\tau_{hg}, \lambda + i\mu_{jk}) \Delta\tau_{hg} \Delta\mu_{jk} - \int_{-r}^r \int_{-b}^b K_{\alpha}(u, v, s, p) \psi(s, p) d\tau d\mu \right\}$$

Now by the definition of seminorm, together with the linearity of the differential operator, the operator  $D_t^l D_x^q$  can be passed through the finite summation and the integral, leading to the following expression as,

$$\sup_{u,v \in E} \left| \sum_{h \rightarrow -g}^{g-1} \sum_{j \rightarrow -k}^{k-1} D_t^l D_x^q K_{\alpha}(u, v, \sigma + i\tau_{hg}, \lambda + i\mu_{jk}) \psi(\sigma + i\tau_{hg}, \lambda + i\mu_{jk}) \Delta\tau_{hg} \Delta\mu_{jk} - \int_{-r}^r \int_{-b}^b \psi(s, p) D_t^l D_x^q K_{\alpha}(u, v, s, p) d\tau d\mu \right|$$

It is observed that  $D_t^l D_x^q$  is continuous. It implies that  $\psi D_t^l D_x^q$  is also continuous.

Hence its Riemann-sum converges to its integral. Therefore, we can write

$$\lim_{g,k \rightarrow \infty} \sum_{h \rightarrow -g}^{g-1} \sum_{j \rightarrow -k}^{k-1} \langle f(u, v), D_t^l D_x^q K_{\alpha}(u, v, \sigma + i\tau_{hg}, \lambda + i\mu_{jk}) \rangle \psi(\sigma + i\tau_{hg}, \lambda + i\mu_{jk}) \Delta\tau_{hg} \Delta\mu_{jk} = \int_{-r}^r \int_{-b}^b \psi(s, p) D_t^l D_x^q K_{\alpha}(u, v, s, p) d\tau d\mu$$

for all  $u, v \in E$ , it follows that for every  $g, k$  the summation is member of  $E$  and it converges in  $E$  to the integral of the R.H.S. of (5.1).

## 5.2 Lemma 2

Statement: For  $\phi(t, x) \in D(I)$ , set  $\psi(s, p)$  as lemma 1 above for  $s, p \in \mathbb{C}^n$ ,  $u$  and  $v$  restricted to compact subset of  $R^n$  then

$$M_{r,b}(u, v) = -\frac{i}{4\pi^2} \int_{-r}^r \int_{-b}^b \psi(s, p) K_{\alpha}(u, v, s, p) d\tau d\mu$$

$$= -\frac{i}{4\pi^2} \int_{-r}^r \int_{-b}^b K_\alpha(u, v, s, p) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{K}_\alpha(t, x, s, p) \phi(t, x) dx dt d\tau d\mu$$

..... (5.2)

converges in  $E$  to  $\phi(t, x)$  as  $r \rightarrow \infty, b \rightarrow \infty$ .

**Proof:** We shall show that  $M_{r,b}(u, v) \rightarrow \phi(u, v)$  in  $E$  as  $r \rightarrow \infty, b \rightarrow \infty$  i.e.

$$\gamma_{E,l,n}[M_{r,b}(u, v) - \phi(u, v)] = \sup_{u,v \in E} |D_t^l D_x^q [M_{r,b}(u, v) - \phi(u, v)]| \rightarrow 0$$

as  $r \rightarrow \infty$  and  $b \rightarrow \infty$ .

We note that for  $l = 0, q = 0$

$$-\frac{i}{4\pi^2} \int_{-r}^r \int_{-b}^b K_\alpha(u, v, s, p) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{K}_\alpha(t, x, s, p) \phi(t, x) dt dx d\tau d\mu = \phi(u, v)$$

This is to say that,

$$\lim_{g,k \rightarrow \infty} M_{r,b}(u, v) = \phi(u, v)$$

since the integrand is a  $C^\infty$  i.e. function of  $u, v$  and  $\phi \in D(I)$ .

We can repeatedly differentiate under the integrand sign in (5.2) and the integrals are uniformly convergent then we have,

$$-\frac{i}{4\pi^2} \int_{-r}^r \int_{-b}^b K_\alpha(u, v, s, p) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{K}_\alpha(t, x, s, p) \phi(t, x) dt dx d\tau d\mu = \phi(u, v)$$

for all  $u, v \in E$ .

Hence proved.

### 5.3 Inversion Theorem

Statement: Let  $f(t, x) \in E'(R^n), 0 < \alpha \leq \frac{\pi}{2}$  and  $\text{supp } f \subset S_{a,b}$  such that

$$S_{a,b} = \{t, x: t, x \in R^n, |t| \leq a, |x| \leq b, a > 0, b > 0\}$$

and let  $F_\alpha(s, p)$  be the distributional fractional Sumudu–Fourier transform of  $f$  is defined as

$$\text{FrSFT}\{f(t, x)\} = F_\alpha(s, p) = \langle f(t, x), K_\alpha(t, x, s, p) \rangle$$

where

$$K_\alpha(t, x, s, p) = \frac{1}{s} \sqrt{\frac{1 - i \cot \alpha}{2\pi i}} e^{\frac{1}{2}[(t^2 + \frac{1}{s^2}) \cot \alpha - \frac{2t}{s} \csc \alpha]} e^{-ipx}$$

then for each  $\phi(t, x) \in E(R^n)$  we have,

$$\lim_{G,H \rightarrow \infty} \left\langle -\frac{i}{4\pi^2} \int_{-G}^G \int_{-H}^H \bar{K}_\alpha(t, x, s, p) F_\alpha(s, p) ds dp, \phi(t, x) \right\rangle = \langle f(t, x), \phi(t, x) \rangle$$

where,

$$\bar{K}_\alpha(t, x, s, p) = \frac{1}{s} \sqrt{\frac{1 + i \cot \alpha}{-2\pi i}} e^{\frac{1}{2}[(t^2 + \frac{1}{s^2}) \cot \alpha - \frac{2t}{s} \csc \alpha]} e^{ipx}$$

Proof: Let  $\phi(t, x) \in E(R^n)$ , we shall show that

$$\left\langle -\frac{i}{4\pi^2} \int_{-r}^r \int_{-b}^b F_\alpha(s, p) \bar{K}_\alpha(t, x, s, p) d\tau d\mu, \phi(t, x) \right\rangle \rightarrow \langle f(t, x), \phi(t, x) \rangle$$

as  $r \rightarrow \infty$  and  $b \rightarrow \infty$ .

..... (5.3)

from the analyticity of  $F_\alpha(s, p)$  on  $C^n$  and the fact that  $\phi(t, x)$  has compact support in  $R$ , it follows that left side

expression in (5.3) is merely a repeated integral with respect to  $t, x, s$  and  $p$ . And the integral in (5.3) is continuous function of  $t, x$  as a closed bounded domain of the integration.

Therefore, we write (5.3) as

$$\begin{aligned} & -\frac{i}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(t, x) \int_{-r}^r \int_{-b}^b F_{\alpha}(s, p) \overline{K_{\alpha}}(t, x, s, p) d\tau d\mu dt dx \\ & -\frac{i}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(t, x) \int_{-r}^r \int_{-b}^b \langle f(u, v), K_{\alpha}(u, v, s, p) \rangle \overline{K_{\alpha}}(t, x, s, p) d\tau d\mu dt dx \\ & -\frac{i}{4\pi^2} \int_{-r}^r \int_{-b}^b \langle f(u, v), K_{\alpha}(u, v, s, p) \rangle \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(t, x) \overline{K_{\alpha}}(t, x, s, p) d\tau d\mu dt dx \end{aligned} \quad \dots\dots\dots (5.4)$$

since  $\phi(t, x)$  is a compact support and the integrand is continuous function of  $(t, x, s, p)$ , the order of integration may be changed. The change in the order of integration is justified by applying to lemma 1.

This yield,

$$-\frac{i}{4\pi^2} \int_{-r}^r \int_{-b}^b \langle f(u, v), K_{\alpha}(u, v, s, p) \rangle \psi(s, p) d\tau d\mu$$

where  $\psi(s, p)$  is in lemma 1. Hence by lemma 1,

$$\begin{aligned} & = \left\langle f(u, v), -\frac{i}{4\pi^2} \int_{-r}^r \int_{-b}^b \overline{K_{\alpha}}(u, v, s, p) \psi(s, p) d\tau d\mu \right\rangle \\ & = \langle f(u, v), \phi(u, v) \rangle \end{aligned}$$

as  $r \rightarrow \infty$  and  $b \rightarrow \infty$ .

( $\because$  by lemma 2)

$$= \langle f(t, x), \phi(t, x) \rangle$$

This is the required proof.

## 6 Uniqueness Theorem

Statement: If  $\{FrSFT[f(t, x)]\}(s, p) = F_{\alpha}(s, p)$  and  $\{FrSFT[g(t, x)]\}(s, p) = G_{\alpha}(s, p)$

for  $0 < \alpha \leq \frac{\pi}{2}$  and  $supp f \subset s_{a,b}$  and  $supp g \subset S_{a,b}$ , where,

$$S_{a,b} = \{t, x: t, x \in R^n, |t| \leq a, |x| \leq b, a > 0, b > 0\}$$

If  $F_{\alpha}(s, p) = G_{\alpha}(s, p)$  then  $f = g$  in the sense of equality in  $E^*(I)$ .

**Proof:** By inversion theorem,

$$\begin{aligned} f(t, x) - g(t, x) &= -\frac{i}{4\pi^2} \int_{-G}^G \int_{-H}^H \overline{K_{\alpha}}(t, x, s, p) [F_{\alpha}(s, p) - G_{\alpha}(s, p)] ds dp \\ &= 0 \end{aligned}$$

Thus,  $f(t, x) = g(t, x)$  in  $E^*(I)$ .

## 7 Conclusion

This study developed the mathematical framework of the Fractional Sumudu–Fourier transform and established its inversion theorem in the setting of testing function space and distribution. The obtain result confirm that the transform is mathematically consistent and allows the original function to be recovered uniquely from its transform form. By combining the advantages of the Sumudu and Fourier transform, the FrSFT provides an efficient tool for analysing function with Fractional and oscillatory behaviour. The result presented in this work may serve as the foundation for future research and application in signal processing, mathematical physics and other areas of applied mathematics.

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